Unique Paper Code	: 42354302_OC
Name of the Course	: B.Sc. (Math Sci)-II/B.Sc. (Phy Sci)-II/ B.Sc. (Life Sci)-II/ B.Sc. (Industrial Chemistry)-II/Analytical Chemistry –II
Name of the Paper	: Algebra-DSC
Semester	: III
Duration	: 3 Hours Maximum Marks: 75

Instructions for Candidates:

1. Attempt any **four** questions out of six questions.
2. All questions carry equal marks.

1. Is the group U (15) cyclic. List all the elements of the group. What is the order of U (15). Check that H= $\left\{\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \mid ad \neq 0; a, b, d \in \mathbb{R}\right\}$ is a normal subgroup of GL (2, $\mathbb{R}$) or not?
Also, determine whether the following permutations are even or odd:
(a) (1 3 5)
(b) (1 3 5 6 7)
(c) (1 2 4 3)(3 5 2 1)

2. Consider the set $\{4, 8, 12, 16\}$. Show that this set is a group under multiplication modulo 20 by constructing its Cayley table. What is the identity element? Is the group cyclic? If so, find all its generators. Show that $\mathbb{Z}_{10}$= ⟨3⟩= ⟨7⟩= ⟨9⟩. Is $\mathbb{Z}_{10}$= ⟨2⟩ ?

3. Let G be a group such that $a^3 = a \; \forall \; a \in G$. Show that G is abelian. Does the converse hold? Justify.
Let $M_2(\mathbb{Q})$ be the ring of 2×2 matrices with entries from $\mathbb{Q}$ (the set of rationals). Let

$$S = \left\{\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{Q}\right\}.$$

Prove or disprove S is a subring of $M_2(\mathbb{Q})$.

Prove that S is commutative with respect to matrix multiplication.

Find the inverse of $\begin{bmatrix} 0 & -b \\ b & 0 \end{bmatrix}$ for $b \neq 0$ in S.

4. Let $R = \mathbb{Z} \oplus \mathbb{Z}$, where $\mathbb{Z} \oplus \mathbb{Z} = \{(a, b) \mid a, b \in \mathbb{Z}\}$ with the operations

$$(a, b) + (c, d) = (a + c, b + d),$$

$$(a, b) \cdot (c, d) = (ac, bd).$$

Find all zero divisors of R. Find $x \neq 0, 1_R$ such that $x^2 = x$.

Let $S = \mathbb{Z}_5 \oplus \mathbb{Z}_6 = \{(a, b) \mid a \in \mathbb{Z}_5, b \in \mathbb{Z}_6\}$. Find the characteristic of S.

Let $H = \{a_0 + a_1 i + a_2 j + a_3 k \mid a_0, a_1, a_2, a_3 \in \mathbb{R}\}$. Is H a field?

Is $\alpha = 1 + i + j + k$ invertible in H? If so, find its inverse β and show that $\alpha\beta = \beta\alpha = 1$.

5. Let V= $\mathbb{R}^3$ and W= $\{(a, b, c) \mid a^2 + b^2 + c^2 \leq 1\}$. Is W a subspace of V ?

Determine whether or not the polynomials $x^2 + 3x - 2$, $2x^2 + 5x - 3$ and $-x^2 - 4x + 4$ generate $P_2(\mathbb{R})$.

Let U be the subspace of $\mathbb{R}^3$ generated by $\{(1,1,0), (1,2,3), (2,3,3)\}$ and W be the subspace of $\mathbb{R}^3$ generated by $\{(1,2,2), (2,3,2), (1,3,4)\}$. Find the dimension of U+W.

6. Let T: $\mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation defined by

T (x, y, z) = (2x+3y-z, x+z)

Find a basis and dimension of (i) R(T), the range space of T (ii) N(T), the null space of T.

Is there a linear transformation T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that

T (x_i)= y_i for i=1, 2, 3

where x_1 = (1, -1), y_1= (1, 0)

x_2 = (2, -1), y_2 = (0, 1)

x_3 = (-3, 2), y_3 = (1, 1)

Unique Paper Code	:	42347902-OC
Name of the Course	:	B.Sc. Programme / B.Sc. Mathematical Science
Name of the Paper	:	Analysis of Algorithms and Data Structures
Semester	:	V
Year of Admission	:	2015/2016/2017/2018

Duration: 3 Hours **Maximum Marks: 75**

Instructions for Candidates

1. All questions carry equal marks.
2. Attempt any **FOUR** Questions.

1 Consider the class Node having info and next fields. Further, the class SinglyLinkedList maintains a single pointer, namely, head and implements the operations defined for maintaining the linked connection of nodes. Determine the functionality of following piece of code:

```
1  function modify()
2      if head != NULL and head.next != NULL
3          prevNode = None
4          nextNode = head.next
5          while head != None
6              head.next = prevNode
7              prevNode = head
8              head = nextNode
9              if head != None
10                 nextNode =head.next
11         head = prevNode
```

Illustrate the step by step execution of the above piece of code for the linked list consisting of elements **13, 19, 5, 20, 18, 25** in sequence. State whether maintaining an extra pointer, namely tail will assist in achieving the above functionality. Justify your answer. Also, write a pseudocode for the doubly and circular linked list achieving the same operation.

2 Consider the following modification in (binary) search algorithm on an array A[1...n], sorted in increasing order. Instead of finding one middle index, we find two middle indices, mid1 and mid2, that divide the array into three equal parts. If an element at mid1 or mid2 equals the element 'x' that we are trying to search, we stop. Else, we do either of the

following: search x from A[1...mid1 -1] if x is less than element at mid1 or search x from A[mid1 +1...mid2 -1] if x is greater than element at mid1 but less than element at mid2 x or search x from A[mid2 +1 ... n] if element at mid2 is less than x.
How will you compute mid1 and mid2? Write the pseudocode for the above-mentioned search strategy. What is the running time of the algorithm?
Consider the array: **-5,-3,1,4,2,-6,1,5**
Can you use Count sort to sort the above array? Justify your answer.

3 Given a binary search tree and the following code to traverse the tree, modify it to count the number of nodes in the tree.

```
fun traverse(Node root)
    if root==NULL
       return
    else
       traverse(left child of root)
       print(value of root)
       traverse(right child of root)
```

Also, determine which of the following statements is/are false about tree Traversals? Justify your answer in each case.

a. If values 1,2,3,4,5,6 are inserted in the given order in a Binary search tree, then in-order, pre-order and level-by-level traversal are all the same.
b. If values 6,5,4,3,2,1 are inserted in the given order in a Binary search tree, then in-order and preorder traversal are the same.
c. If values 6,5,4,3,2,1 are inserted in the given order in a Binary search tree, then pre-order and level-by-level traversal are the same.
d. If values 6,5,4,3,2,1 are inserted in the given order in a Binary search tree, then in-order and postorder traversal are the same.

4 Consider the infix expression: **A+B/C-(D-E)$F*G** (where $ stands for exponentiation). Represent the given infix expression using a binary tree. Identify the ways in which the constructed binary tree can be traversed and write the traversal sequence for each one of them.

Suppose, we wish to represent the infix expression in postfix notation. Identify the data structure that you will employ for this conversion. Show the step-by-step sequence of operations involved. Further, determine the data structure that you will use for evaluating the postfix notation. Using the identified data structure and the values **A=5, B=6, C=3, D=5, E=3, F=2, G=3**; evaluate the postfix expression.

5 Which data structure would you choose for the following tasks and why?
Justify your answer for each case.

a. To implement undo/redo operation in a text editor.
b. Bookmarks tab of a web-browser.
c. You are looking for a way out of a maze and you are not allowed to use recursion. You have to store the path that you are currently exploring and be able to go back one step whenever you find yourself in a dead-end and explore a new possibility from there.
d. You want to store a sorted list of strings and support the merging of two sorted lists into one, in-place (i.e., without creating a copy of the lists).
e. You are writing software for a call center. When a client calls, his call should be stored until there is a free operator to pick it up. Calls should be processed in the same order in which they were received.
f. You want to store the stations of a public transportation line. New stations can be added at both ends of the line but not between existing stations. You should be able to traverse the line in both directions.
g. You need to create a table of contents, i.e., the index page of a book. It must show every chapter, its sections and corresponding sub-sections, to all levels.
h. You need to create a glossary for a book. It must show important terms in sorted order. Synonyms or inter-related terms must have each other's reference.

6 Consider the following piece of code:

```
1 function enigma(integer n)
2       if (n>0)
3           enigma(n-2)
4           print(n)
5           enigma(n-2)
6       else
7           print('*')
8
9 function main()
10 enigma(7)
11
12 main()
```

Determine the output of above code. How many times will enigma(n) be called to determine the value of enigma(7)? Draw the tree showing all the calls generated by enigma(7). Which data structure is used for tracking each recursive function call?
Determine the output for the function call enigma(7) after eliminating the statement in line 5.
Further, write the iterative version for this modified code (after eliminating statement in line 5), and derive its time complexity, give reasoning for the answer.